

Instructions to the Students:

1. All the questions are compulsory.
2. The level of question/expected answer as per OBE or the Course Outcome (CO) on which the question is based is mentioned in () in front of the question.
3. Use of non-programmable scientific calculators is allowed.
4. Assume suitable data wherever necessary and mention it clearly.

(Level/ Marks
CO)

Q. 1 Attempt the following questions.

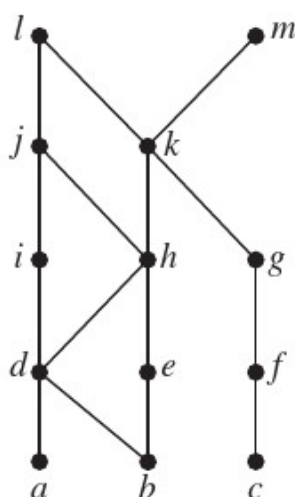
[12]

- A)** Let $P(x)$, $Q(x)$, and $R(x)$ be the statements “ x is a professor,” “ x is ignorant,” and “ x is vain,” respectively. Express each of these statements using quantifiers; logical connectives; and $P(x)$, $Q(x)$, and $R(x)$, where the domain consists of all people. **CO1**
- (a) No professors are ignorant. (b) All ignorant people are vain.
- (c) No professors are vain. (d) Does (c) follows (a) and (b)?
- B)** Show that if n is a positive integer, then $1+2+3+\dots+n=\frac{n(n+1)}{2}$ by using mathematical induction. **CO1**
- C)** Use De Morgan’s laws to find the negation of each of the following statements. **CO1**
- (a) Kwame will take a job in industry or go to graduate school.
- (b) Yoshiko knows Java and calculus.
- (c) James is young and strong.
- (d) Rita will move to Oregon or Washington.

Q.2 Solve Any Two of the following.

[12]

- A)** Answer these questions for the partial order represented by this Hasse diagram. **CO2**



- a) Find the maximal elements.
- b) Find the minimal elements.
- c) Is there a greatest element?
- d) Is there a least element?
- e) Find all upper bounds of $\{a, b, c\}$.
- f) Find all lower bounds of $\{f, g, h\}$.

B) Let $f(x) = x + 2$, $g(x) = x - 2$, and $h(x) = 3x$ for $x \in \mathbb{R}$ where $\mathbb{R}$ = set of real numbers. Find **C02**

i) gof

ii) fohog

iii) foh

C) Solve the following homogeneous recurrence relation: **C03**

$$t_n = 4t_{n-1} - 4t_{n-2}.$$

$$t_n = 1, n = 0 \text{ and } n = 1.$$

Q. 3 Solve Any Two of the following.

[12]

A) Draw the following graphs. **C04**

a) K_7

b) $K_{1,8}$

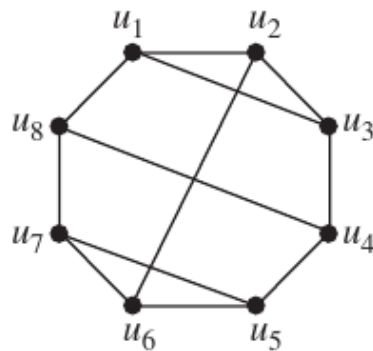
c) $K_{4,4}$

d) C_7

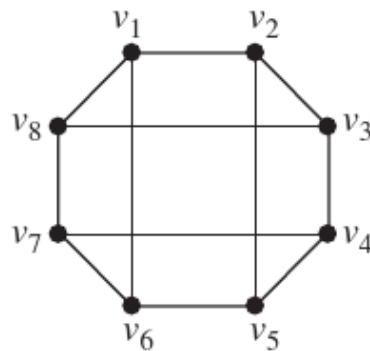
e) W_7

f) Q_4

B) Use paths either to show that these graphs are not isomorphic or to find an isomorphism between them. **C04**

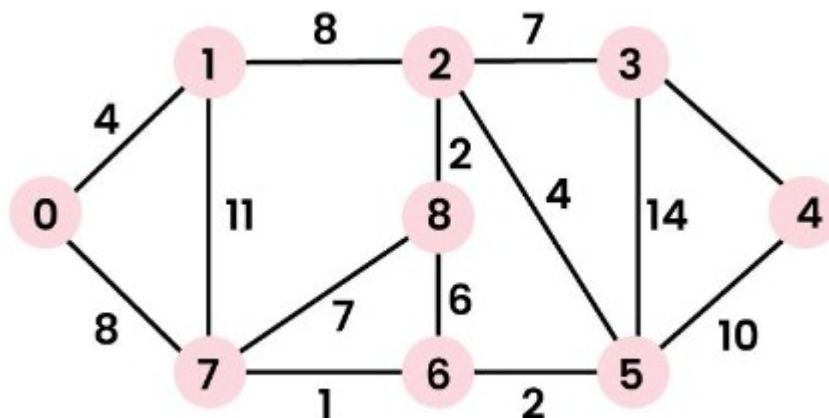


G



H

C) Use Dijkstra's algorithm to find the length of a shortest path between the vertices 1 and 4 in the weighted graph displayed in Figure below. **C04**



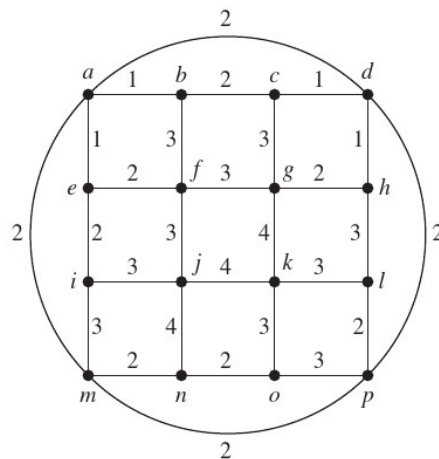
Q.4 Attempt the following questions.

[12]

A) Use Huffman's algorithm to provide an optimal average-bit-length code for the probability distribution a: 0.2, b: 0.2, c: 0.15, d: 0.15, e: 0.15, f: 0.1, and g: 0.05. Draw a binary tree to find the prefix code for each probability distribution **C04**

and compute the average bit-length of a codeword.

- B)** Use Kruskal's algorithm to find a minimum spanning tree for the weighted graph in **C04** the Figure below.



- C)** (i) What is the value of each of the prefix expressions $+ - \uparrow 3 2 \uparrow 2 3 / 6 - 4 2$? **C04**
(ii) What is the value of each of the postfix expressions $3 2 * 2 \uparrow 5 3 - 8 4 / * -$?

Q. 5 Attempt the following questions.

[12]

- A)** Consider the binary operation defined on the set $A = \{a, b, c, d\}$ by following table. **C05**
Find:

*	a	b	c	d
a	a	c	b	d
b	d	a	b	c
c	c	d	a	a
d	d	b	a	c

- (i) $c * d$ and $d * c$
(ii) $b * d$ and $d * b$
(iii) $a * (b * c)$ and $(a * b) * c$
- B)** Consider the group $G = \{1, 2, 4, 7, 8, 11, 13, 14\}$ under multiplication modulo 15. **C05**
i) Find multiplication table of G .
ii) Find 2^{-1} , 7^{-1} .
iii) Find the orders and subgroups generated by 7 and 11.

*** End ***